Oxford Cambridge and RSA

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Question Set 1

1

Write down the value of

(A) $\log_a(a^4)$, $= 4\log_a a = 4$ [1]

(B) $\log_a\left(\frac{1}{a}\right)$. $= -\log_a a = -1$ [1]

2

P and Q are consecutive **odd** positive integers such that $P > Q$.

Prove that $P^2 - Q^2$ is a multiple of 8. $(2n+1)^2 - (2n-1)^2$ [3]

$= 4n^2 + 4n + 1 - 4n^2 + 4n - 1 = 8n \therefore$ multiple of 8

3

Find the set of values of a for which the equation

$$ax^2 + 8x + 2 = 0$$

has no real roots. [3]

$8^2 - 4 \times 2a < 0$

$64 < 8a$

$8 < a$

4

Show that $\int_0^9 (3 + 4\sqrt{x})\,dx = 99$ [4]

$$\left[3x + \frac{8}{3}x^{3/2}\right]_0^9 = (27 + 72) - (0) = 99 - 0 = 99$$

5

In this question you must show detailed reasoning

The centre of a circle C is at the point (−1, 3) and C passes through the point (1, −1). The straight line L passes through the points (1, 9) and (4, 3). Show that L is a tangent to C.

CIRCLE:

Centre: $(x+1)^2 + (y-3) = r^2$

$(1, -1)$ $(1+1)^2 + (-1-3)^2 = r^2$

$2^2 + (-4)^2 = r^2$

$20 = r^2$

$\sqrt{20} = r$

LINE L:

gradient $= \frac{3-9}{4-1} = -2$

equation: $y = -2x + c$

substitute in: $9 = -2(1) + c$

$11 = c$

$\therefore y = -2x + 11$

Show that L is a tangent to C:

$y = -2x + 11$

$(x+1)^2 + (-2x + 11 - 3)^2 = 20$

$(x+1)^2 + (-2x + 8)^2 = 20$

$x^2 + 2x + 1 + 4x^2 - 32x + 64 = 20$

$5x^3 - 30x + 45 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$\therefore$ repeated root at

$x = 3$

$\therefore$ tangent

6 **(i)**

A curve has equation $y = 16x + \frac{1}{x^2}$. Find $y = 16x + x^{-2}$

(A) $\frac{dy}{dx} = 16 - 2x^{-3} = 16 - \frac{2}{x^3}$ [2]

(B) $\frac{d^2y}{dx^2} = +6x^{-4} = \frac{6}{x^4}$ [2]

(ii) Hence

- find the coordinates of the stationary point,
- determine the nature of the stationary point. [5]

co-ordinates:

x: $\frac{dy}{dx} = 0$, $16 - \frac{2}{x^3} = 0$

$16 = \frac{2}{x^3}$

$16x^3 = 2$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

y: $y = 16(\frac{1}{2}) + \frac{1}{(\frac{1}{2})^2}$

$= 8 + 4 = 12$ $\therefore (\frac{1}{2}, 12)$

Determine the nature of the stationary point.

$\frac{d^2y}{dx^2} = \frac{6}{x^4} = \frac{6}{(\frac{1}{2})^4} = \frac{6}{1/16} = 96$

$96 > 0$, therefore it is a minimum point.

7

In an experiment 500 fruit flies were released into a controlled environment. After 10 days there were 650 fruit flies present.

Munirah believes that N, the number of fruit flies present at time t days after the fruit flies are released, will increase at the rate of 4.4% per day. She proposes that the situation is modelled by the formula $N = Ak^t$.

(i) Write down the values of A and k. [2]

When $t = 0$, $N = A$ and rate of increase $= 4.4\%$.

$A = 500$

$k = 1.044$

(ii) Determine whether the model is consistent with the value of N at $t = 10$. [2]

$N = 500 \times 1.044^t$

@ $t = 10$ [1]

$N = 769.$

$769 \neq 650$ $\therefore$ not consistent

(iii) What does the model suggest about the number of fruit flies in the long run? [1]

It will always increase

Subsequently it is found that for large values of t the number of fruit flies in the controlled environment oscillates about 750. It is also found that as t increases the oscillations decrease in magnitude.

Munirah proposes a second model in the light of this new information.

$$N = 750 - 250 \times e^{-0.092t}.$$

(iv) Identify three ways in which this second model is consistent with the known data. [3]

1. originally is equal to 500
2. = 650 at t = 10
3. long term value is 750

(v) **(A)** Identify one feature which is not accounted for by the second model. Oscillations [1]

(B) Give an example of a mathematical function which needs to be incorporated in the model to account for this feature. [1]

Trig function

E.g cos function

Total Marks for Question Set 1: 38 marks